# Montana Common Core Standards and Assessments 

## Montana Curriculum Organizer

## Grade 4

## Mathematics



Montana
Office of Public Instruction
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opi.mt.gov

## Montana Curriculum Organizer: Grade 4 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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## HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 23 consist of tables organized into learning progressions that can function as units. The table for each learning progression unit includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

| LEARNING PROGRESS | STANDARDS IN LEARNING PROGRESSION |
| :---: | :---: |
| Name of this learning progression, often with a domain, however in some cases or combined. | Standards covered in this learning progression. |
| UNDERSTAND: |  |
| What students need to understand by the end of this learning progression. |  |
| KNOW: | DO: |
| What students need to know by the end of this learning progression. | need to be able to do by the end of this learning progression, ster and standard. |
| KEY TERMS FOR THIS PROGRESSION: |  |
| Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here. |  |
| INSTRUCTIONAL STRATEGIES AND RESOURCES: |  |
| Cluster: Title <br> Strategies for this cluster <br> Instructional Resources/Tools <br> Resources and tools for this cluster |  |
| Cluster: Title <br> Strategies for this cluster <br> Instructional Resources/Tools <br> Resources and tools for this cluster |  |
| CONNECTIONS TO OTHER DOMAINS ANDIOR CLUSTERS: |  |
| Standards that connect to this learning progression are listed here, organized by cluster. |  |
| STANDARDS FOR MATHEMATICAL PRACTICE: |  |
| A quick reference guide to the eight standards for mathematical practice is listed here. |  |

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Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

| STANDARDS FOR MATHEMATICAL PRACTICE: GRADE 4 EXPLANATIONS AND EXAMPLES |  |
| :---: | :---: |
| Standards | Explanations and Examples |
| Students are expected to: | The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. |
| 4.MP.1. Make sense of problems and persevere in solving them. | In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourthgraders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers. |
| 4.MP.2. Reason abstractly and quantitatively. | Fourth-graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place-value concepts. |
| 4.MP.3. Construct viable arguments and critique the reasoning of others. | In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. |
| 4.MP.4. Model with mathematics. | Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth-graders should evaluate their results in the context of the situation and reflect on whether the results make sense. |
| 4.MP.5. Use appropriate tools strategically. | Fourth-graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. |
| 4.MP.6. Attend to precisio | As fourth-graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot. |
| 4.MP.7. Look for and make use of structure. | In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule. |
| 4.MP.8. Look for and express regularity in repeated reasoning. | Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions. |

## CRITICAL AREAS FOR GRADE 4 MATH

In Grade 4, instructional time should focus on three critical areas:
(1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends;
(2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and
(3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

| LEARNING PROGRESSION | STANDARDS IN LEARNING PROGRESSION |
| :--- | :--- |
| Operations \& Algebraic Thinking | 4.OA.1, 4.OA.2, 4.OA.3, 4.OA.4, 4.OA.5 |
| UNDERSTAND: |  |

Factors and multiples can be used to determine part/whole relationships.
By utilizing efficient methods of multiplication and division, more complex problem solving is possible.

| KNOW: | DO: |
| :---: | :---: |
| Multiplication scenarios can be interpreted differently based on the context of the problem. (e.g., a " 5 times greater than 7 " problem is interpreted differently than " 5 groups of 7 " but both are derived from $5 \times 7$ ). <br> Additive thinking is "How many more?" <br> Multiplicative thinking is "How many times more?" <br> Problems can be solved by writing the solution pathway in algebraic notation and then solving for the unknown. <br> Estimation in multiplication and division can predict the size of the answer and help to | Use the four operations with whole numbers to solve problems. <br> 4.OA.1 Interpret a multiplication equation as a comparison (e.g., interpret $35=5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. <br> 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison. ${ }^{1}$ <br> 4.OA. 3 Solve multistep word problems within cultural contexts, including those of Montana American Indians, posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |

## Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

## Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

## KEY TERMS FOR THIS PROGRESSION:

Composite, Equation, Estimation, Factors, Multiples, Prime

## INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use the four operations with whole numbers to solve problems.
Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Table 2 on page 73 in the Montana Common Core Standards for School Mathematics Grade-Band. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or

[^0]additive comparison (solved when adding and subtracting in Grades 1 and 2).
Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Examples of multistep word problems can be accessed from the released questions on the National Assessment of Educational Progress. (NAEP) Assessment. For example, a constructed response question from the 2007 Grade 4 NAEP assessment reads, "Five classes are going on a bus trip and each class has 21 students. If each bus holds only 40 students, how many buses are needed for the trip?"

## Instructional Resources/Tools

Institute of Education Sciences. National Center for Education Statistics. National Assessment of Educational Progress (NAEP) Assessments.

Montana Office of Public Instruction. 2011. Montana Common Core Standards for School Mathematics Grade-Band. Page 73, Table 2.

## Cluster: Gain familiarity with factors and multiples.

Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counter-examples of factors.

Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole-number factor that is not one or itself. Starting with a number chart of 1 to 20 , use multiples of prime numbers to eliminate later numbers in the chart (e.g., 2 is prime, but $4,6,8,10,12, \ldots$ are composite). Encourage the development of rules that can be used to aid in the determination of composite numbers (e.g., other than 2 , if a number ends in an even number ( $0,2,4,6$ and 8 ), it is a composite number).

Using area models will also enable students to analyze numbers and arrive at an understanding of whether a number is prime or composite. Have students construct rectangles with an area equal to a given number. They should see an association between the number of rectangles and the given number for the area as to whether this number is a prime or composite number.

Definitions of prime and composite numbers should not be provided, but determined after many strategies have been used in finding all possible factors of a number.

Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets (e.g., have them find several factors of $10,14,25$ or 32 , and write multiplication expressions for the numbers).

Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. For example if you have 16 squares:


The idea that a product of any two whole numbers is a common multiple of those two numbers is a difficult concept to understand. For example, $5 \times 8$ is 40 ; the table below shows the multiples of each factor.

| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |

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Ask students what they notice about the number 40 in each set of multiples; 40 is the $8^{\text {th }}$ multiple of 5 , and the $5^{\text {th }}$ multiple of 8.
Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in Grade 6.
Writing multiplication expressions for numbers with several factors and for numbers with a few factors will help students in making conjectures about the numbers. Students need to look for commonalities among the numbers.

## Instructional Resources/Tools

Calculators
Counters
Grid papers
Drexel University. The Math Forum. 1994-2012. Understanding factoring through geometry: Using square unit tiles, students work with a partner to construct all rectangles whose area is equal to a given number. After several examples, students see that prime numbers are associated with exactly two rectangles, whereas composite numbers are associated with more than two rectangles.

National Council of Teachers of Mathematics. 2000-2012.
Factor Game: Engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions of prime and composite numbers.
Multiplication: It's in the Cards: More Patterns with Products: Students practice multiplication facts and record their current level of mastery on their personal multiplication chart.
The Product Game: Students start with factors and multiply to find the product.
The Product Game - Classifying Numbers: Students construct Venn diagrams to show the relationships between the factors or products of two or more numbers in the Product Game.

Utah State University. 1999-2000.
National Library of Virtual Manipulatives: The National Library of Virtual Manipulatives contains Java applets and activities for K-12 mathematics.
Sieve of Eratosthenes: relate number patterns with visual patterns. Click on the link for Activities for directions on engaging students in finding all prime numbers 1-100.
Cluster: Generate and analyze patterns.
In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to "undo" the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, "Add 4 " starting with the number 1 , the pattern $1,5,9$, $13,17, \ldots$ is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, "How is a number in the sequence related to the one that came before it?", and "If they started at the end of the pattern, will this relationship be the same?" Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Students should also determine if there are other relationships in the patterns. In the numeric pattern generated above, students should observe that the numbers are all odd numbers.

Provide patterns that involve shapes so that students can determine the rule for the pattern. For example, students may state that the rule is to multiply the previous number of squares by 3.


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Instructional Resources/Tools
National Council of Teachers of Mathematics. 2000-2012.
Patterns that Grow: Looking Back and Moving Forward: In this final lesson of the unit, students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts.
Patterns that Grow: Growing Patterns: Students use numbers to make growing patterns. They create, analyze, and describe growing patterns and then record them. They also analyze a special growing pattern called Pascal's triangle.

Public Broadcasting Service. 1995-2012. Snake Patterns-s-s-s: Students will use given rules to generate several stages of a pattern and will be able to predict the outcome for any stage.

## CONNECTIONS TO OTHER DOMAINS ANDIOR CLUSTERS:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.MD.1, 4.MD.2, 4.MD.3)

Generalize place-value understanding for multi-digit whole numbers. (4.NBT.1, 4.NBT.2)
Use place-value understanding and properties of operations to perform multi-digit arithmetic. (4.NBT.4)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NF.4)
Understand decimal notation for fractions, and compare decimal fractions. (4.NF.5, 4.NF.7)

## STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

| LEARNING PROGRESSION | STANDARDS IN LEARNING PROGRESSION |
| :---: | :---: |
| Number and Operations in Base Ten ${ }^{2}$ - Place Value | lue $\quad$ 4.NBT.1, 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.NBT.6 |
| UNDERSTAND: |  |
| The number system is a repeated counting pattern based on tens and powers of ten. Efficient strategies for multi-digit arithmetic are based on applying the properties of operations. |  |
| KNOW: | DO: |
| Expanded notation can be used to show order, values of each digit, and the powers of 10 . <br> The Distributive Property of Multiplication can be modeled in an array as well as with expanded notation. <br> Rounding a number to the largest place value can be accomplished by answering: "Is this number closest to N -thousand or $\mathrm{N}+1$ thousand?" <br> Multiplication and division are inverse operations. | Generalize place-value understanding for multi-digit whole numbers. <br> 4.NBT. 1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70=10$ by applying concepts of place value and division. <br> 4.NBT. 2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. <br> 4.NBT. 3 Use place-value understanding to round multi-digit whole numbers to any place. <br> Use place-value understanding and properties of operations to perform multi-digit arithmetic. <br> 4.NBT. 4 Fluently add and subtract multi-digit whole numbers using the standard algorithm. <br> 4.NBT. 5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. <br> 4.NBT. 6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
| KEY TERMS FOR THIS PROGRESSION: |  |
| Expanded Form (1,000+200+30+4), Place value, Standard Form (1,234), Word Form (One thousand, twohundred thirty-four) |  |

## INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Generalize place-value understanding for multi-digit whole numbers.
Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multidigit whole numbers.

Students need to have opportunities to compare numbers with the same number of digits (e.g., compare 453, 698 and 215); numbers that have the same number in the leading digit position (e.g., compare 45,495 and 41,223); and numbers that have different numbers of digits and different leading digits (e.g., compare 312, 95, 5,245 and 10,002).

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select four to six cards; then, using all the cards, make the largest number possible with the cards, the smallest number possible and the closest number to 5,000 that is greater than 5,000 or less than 5,000.

In Grade 4, rounding is not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value. What is new for Grade 4 is rounding to digits other than the leading digit (e.g.,

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round 23,960 to the nearest hundred). This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1,000, not just zero.

Students should also begin to develop some rules for rounding, building off the basic strategy of "Is 48 closer to 40 or 50 ?" Since 48 is only 2 away from 50 and 8 away from 40,48 would round to 50 . Now students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit.

## Instructional Resources/Tools

Number cards
Place-value boxes
Place-value flip charts
Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.
A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about; rather than merely follow a sequence of directions that they don't understand.

It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to understand all the steps in the algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. "Think-Pair-Share" is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers, provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even attempt to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (e.g., multiples of 10) when doing mental math, and this promotes its understanding.

Use a variation of an area model. For example, to multiply $23 \times 36$, arrange the partial products as follows:

|  | $20+3$ |  |
| :---: | :--- | :--- |
| $30+6$ | 600 | 90 |
|  | 120 | 18 |

Then add the four partial products to get 828 .

As students developed an understanding of multiplying a whole number up to four digits by a one-digit whole number, and multiplying two two-digit numbers through various strategies, they should do the same when finding whole-number quotients and remainders. By relating division to multiplication and repeated subtraction, students can find partial quotients. An explanation of partial quotients can be viewed during this video on TeacherTube.com: Outline of partial quotients. This strategy will help them understand the division algorithm.

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Students will have a better understanding of multiplication or division when problems are presented in context.
Students should be able to illustrate and explain multiplication and division calculations by using equations, rectangular arrays and the properties of operations. These strategies were used in Grade 3 as students developed an understanding of multiplication.

To give students an opportunity to communicate their understandings of various strategies, organize them into small groups and ask each group to create a poster to explain a particular strategy and then present it to the class.

Vocabulary is important. Students should have an understanding of terms such as, sum, difference, fewer, more, less, ones, tens, hundreds, thousands, digit, whole numbers, product, factors and multiples.

## Instructional Resources/Tools

Base-ten blocks
Bound place-value flip books (so that the digit in a certain place can be switched)
Hundreds flats
Place-value mats
Smartboard
Tens frames
TeacherTube.com. Outline of partial quotients. An explanation of partial quotients can be viewed during this video.
Whitin, David. Read Any Good Math Lately? 1992. A resource book that makes a connection to literature. This will help to identify books related to certain math topics. Books can provide a 'hook' for learning, to activate background knowledge, and to build student interest.

## CONNECTIONS TO OTHER DOMAINS ANDIOR CLUSTERS:

Understand decimal notation for fractions, and compare decimal fractions. (4.NF.5, 4.NF.6, 4.NF.7)

## STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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| LEARNING PROGRESSION | STANDARDS IN LEARNING PROGRESSION |
| :---: | :---: |
| Number and Operations - Fractions - Equivalence, Comparing Fractions \& Decimals | e, $\begin{aligned} & \text { 4.NF.1, 4.NF.2, 4.NF.5, 4.NF.6, 4.NF.7 }\end{aligned}$ |
|  | UNDERSTAND: |
| Equivalent fractions or decimal fractions represent the same quantity in multiple ways. Using visual models and place value is helpful in comparing fractions and decimals. |  |
| KNOW: | D |
| Multiplying a fraction by one always results in an equivalent fraction (e.g., $1 / 4 \times 3 / 3=3 / 12$ ). <br> Equivalent fractions can be generated using area models, ratio models, number lines and fractions bars. <br> Compare fractions using common denominator, common numerator, comparison to benchmark and distance to benchmark; as well as determining when each strategy is appropriate. <br> Compare decimal fractions using $10 \times 10$ grid, a number line, and measurement such as metric system, money. | Grade 4 expectations in this domain are limited to fractions with denominators of $2,3,4,5,6,8,10,12$, and 100. <br> Extend understanding of fraction equivalence and ordering. <br> 4.NF. 1 Explain why a fraction $a / b$ is equivalent to a fraction $(n \times a) /(n \times$ b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. <br> 4.NF. 2 Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators), or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model). <br> Understand decimal notation for fractions, and compare decimal fractions. <br> 4.NF. 5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and $100 .{ }^{3}$ For example, express $3 / 10$ as $30 / 100$, and add $3 / 10+4 / 100=34 / 100$. <br> 4.NF. 6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. <br> 4.NF. 7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, (e.g., by using a visual model). |
| KEY TERMS FOR THIS PROGRESSION: |  |
| Benchmark fractions, Denominators, Equivalent fractions, Hundredth, Numerators, Tenth |  |
| INSTRUCTIONAL STRATEGIES AND RESOURCES: |  |
| Cluster: Extend understanding of fractions equivalence and ordering. <br> Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions. <br> Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators. <br> Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular |  |
|  |  |
|  |  |

[^2]models) so that the models represent the same whole. The models should be represented in drawings. Students should also use benchmark fractions such as $1 / 2$ to compare two fractions. The result of the comparisons should be recorded using >, < and = symbols.


## Instructional Resources/Tools

Fraction bars or strips
Pattern blocks
Cluster: Understand decimal notations for fractions, and compare decimal fractions.
The place-value system developed for whole numbers extends to fractional parts represented as decimals. This is a connection to the metric system. Decimals are another way to write fractions. The place-value system developed for whole numbers extends to decimals. The concept of one whole used in fractions is extended to models of decimals.

Students can use base-ten blocks to represent decimals. A $10 \times 10$ block can be assigned the value of one whole to allow other blocks to represent tenths and hundredths. They can show a decimal representation from the base-ten blocks by shading on a $10 \times 10$ grid.

Students need to make connections between fractions and decimals. They should be able to write decimals for fractions with denominators of 10 or 100 . Have students say the fraction with denominators of 10 and 100 aloud. For example, $4 / 10$ would be "four tenths" or 27/100 would be "twenty-seven hundredths." Also, have students represent decimals in word form with digits and the decimal place value, such as 0.4 would be 4 tenths.

Students should be able to express decimals to the hundredths as the sum of two decimals or fractions. This is based on understanding of decimal place value. For example 0.32 would be the sum of 3 tenths and 2 hundredths. Using this understanding students can write 0.32 as the sum of two fractions $(3 / 10+2 / 100)$.

Students' understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5.

In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations before they try to do conversions in the metric system. Understanding of the decimal place-value system is important prior to the generalization of moving the decimal point when performing operations involving decimals.


Students extend fraction equivalence from Grade 3 with denominators of 2, 3, 4, 6 and 8 to fractions with a denominator of 10. Provide fraction models of tenths and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a $10 \times 10$ grid. The $10 \times 10$ grid is defined as one whole. The decimal must relate to the whole.

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0.3

0.03

Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

Instructional Resources/Tools
$10 \times 10$ square on a grid
Base-ten blocks
Decimal place-value mats
Length or area models
Number lines
National Council of Teachers of Mathematics. 2000-2012.A Meter of Candy: In this series of three hands-on activities, students develop and reinforce their understanding of hundredths as fractions, decimals and percentages. Students explore with candy pieces as they physically make and connect a set and linear model (meter) to produce area models (grids and pie graphs). At this time, students are not to do percents. The relationships among fractions, decimals and percents are developed in Grade 6.

## CONNECTIONS TO OTHER DOMAINS ANDIOR CLUSTERS:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.MD.1, 4.MD.2)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.
3. Attend to precision.
4. Construct viable arguments and critique the reasoning of others.
5. Model with mathematics.
6. Look for and make use of structure.
7. Look for and express regularity in repeated reasoning.

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STANDARDS IN LEARNING PROGRESSION:
4.NF.3a-d, 4.NF.4a-c

## UNDERSTAND:

Fractions are built from unit fractions through the process of addition and multiplication.
Visual fraction models and equations are tools for adding fractions, subtracting fractions, and multiplying a fraction by a whole number

A fraction $a / b$ is a multiple of $1 / b$ (i.e., a groups of $1 / b=a \times 1 / b$ ). For example, $5 / 4$ is the same as 5 sets of $1 / 4$ or $5 \times 1 / 4$.

A mixed number is the sum of its decomposed fractional parts. For example, $2 \frac{1}{4}={ }^{4} / 4+{ }^{4} / 4+$ $1 / 4$.

Decomposing $3 / 4$ into $1 / 4+1 / 4+1 / 4$ allows for adding or subtracting fourths.

Either factor can be the multiplier when
multiplying a fraction by a whole number. For example, $1 / 2 \times 6$ or $6 \times 1 / 2$

Visual Fraction models:
Area model
Array
Clock model
Fraction bars
Number line

## DO:

Grade 4 expectations in this domain are limited to fractions with denominators of $2,3,4,5,6,8,10,12$, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF. 3 Understand a fraction $a / b$ with $a>1$ as a sum of fractions $1 / b$.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, by using a visual fraction model (e.g., $3 / 8=1 / 8+1 / 8+1 / 8 ; 3 / 8=$ $1 / 8+2 / 8 ; 2^{1 /} / 8=1+1+1 / 8=8 / 8+8 / 8+1 / 8$.
c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction), and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems within cultural contexts, including those of Montana American Indians, involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).
4.NF. 4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction $a / b$ as a multiple of $1 / b$. For example, use a visual fraction model to represent 5/4 as the product $5 \times$ (1/4), recording the conclusion by the equation $5 / 4=5 \times(1 / 4)$.
b. Understand a multiple of $a / b$ as a multiple of $1 / b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times(2 / 5)$ as $6 \times(1 / 5)$, recognizing this product as 6/5. (In general, $n \times$ $(a / b)=(n \times a) / b$.
c. Solve word problems within cultural contexts, including those of Montana American Indians, involving multiplication of a fraction by a whole number (e.g., by using visual-fraction models and equations to represent the problem). For example, if each person at a party will eat $3 / 8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Decompose, Equivalence, Equivalent, Fractions, Justify, Mixed numbers

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## INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
In Grade 3, students added unit fractions with the same denominator. Now, they begin to represent a fraction by decomposing the fraction as the sum of unit fraction and justify with a fraction model. For example, $\frac{3}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$.


Students also represented whole numbers as fractions. They use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem.

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawings to show their understanding.

Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

## Instructional Resources/Tools

Circular fraction models
Fraction tiles/bars
Number lines
Rulers with markings of $1 / 2,1 / 4$, and $1 / 8$.

## CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.MD.2)

Represent and interpret data. (4.MD.4)

## STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the
4. Use appropriate tools strategically.
reasoning of ohers.
5. Attend to precision. reasoning of others.
6. Look for and make use of structure.
7. Look for and express regularity in repeated reasoning.

| LEARNING PROGRESSION: |  | STANDARDS IN LEARNING PROGRESSION: |
| :---: | :---: | :---: |
| Measurement and Data - Measurement Sy Perimeter, Data |  | 4.MD.1, 4.MD.2, 4.MD.3, 4.MD.4 |
| UNDERSTAND: |  |  |
| Within a single system of measurement larger units are made from smaller units (i.e., $1 \mathrm{~km}=1,000$ meters). Smaller units are divisions of larger unit (i.e., $1 \mathrm{~cm}=1 / 100$ of a meter). <br> Formulas are an efficient way to solve for area and perimeter. <br> Line plots can be used to represent data. |  |  |
| KNOW: | DO: |  |
| Relative sizes of measurement units (i.e., km, cm, kg, g, lb., oz., liter, ml, min., sec., hr.). <br> Equivalent measurements within a measurement system can be used to solve problems (e.g., $4 \mathrm{~m}=400 \mathrm{~cm}$, and $24 \mathrm{in}=2$ ft.). <br> An array model can justify the formulas: $A=L x$ $W$ and $P=2 L+2 W$. <br> Line plots with whole numbers must include all the whole numbers in the range. <br> Line plots with fractions must include all whole numbers and fractions within the range (e.g.; 3, $3 \frac{1}{2}, 4,4 \frac{1}{2}$, etc.). <br> Consistent increments. | Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. <br> 4.MD. 1 Know relative sizes of measurement units within one system of units including km, m, cm, kg, g, lb., oz., l, ml, hr., min., and sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1,12),(2,24),(3,36)$, etc. <br> 4.MD. 2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. <br> 4.MD. 3 Apply the area and perimeter formulas for rectangles in realworld and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. <br> Represent and interpret data. <br> 4.MD. 4 Make a line plot to display a data set of measurements in fractions of a unit (e.g., 1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. |  |
| KEY TERMS FOR THIS PROGRESSION: |  |  |
| Area, Line plot, Measurement, Perimeter |  |  |
| INSTRUCTIONAL STRATEGIES AND RESOURCES: |  |  |
| Cluster: Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit. <br> In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class. The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard. Allow students to use rulers and yardsticks to discover these relationships among these units of measurements. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers, which is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick. Have students record the relationships in a two-column table or a T-chart. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters. <br> Present word problems as a source of students' understanding of the relationships among inches, feet and yards. Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. |  |  |

Present problems that involve multiplication of a fraction by a whole number (denominators are $2,3,4,56,8,10,12$ and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts.

Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems.

## Instructional Resources/Tools

Graduated measuring cups (marked with customary and metric units)
Teaspoons and tablespoons
Yardsticks (meter sticks) and rulers (marked with customary and metric units)

## Cluster: Represent and interpret data.

Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit (1/2, 1/4, 1/8) and plot data showing multiple data points for each fraction.


Pose questions that students may answer, such as:

- "How many one-eighths are shown on the line plot?" Expect "two one-eighths" as the answer. Then ask, "What is the total of these two one-eighths?" Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.
- "What is the total number of inches for insects measuring 38 inches?" Students can use skip counting with fraction names to find the total, such as, "three-eighths, six-eighths, nine-eighths". The last fraction names the total. Students should notice that the denominator did not change when they were saying the fraction name. Have them make a statement about the result of adding fractions with the same denominator.
- "What is the total number of insects measuring 18 inch or 58 inches?" Have students write number sentences to represent the problem and solution such as, $18+18+58=78$ inches.

Use visual-fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

## Instructional Resources/Tools

Fraction bars or strips
CONNECTIONS TO OTHER DOMAINS ANDIOR CLUSTERS:
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NF.3)
Understand decimal notation for fractions, and compare decimal fractions. (4.NF.6, 4.NF.7)

## STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

| LEARNING PROGRESSION: | STANDARDS IN LEARNING PROGRESSION: |
| :---: | :---: |
| Geometry \& Angle Measurement | 4.MD.5a-b, 4.MD.6, 4.MD.7, 4.G.1, 4.G.2, 4.G. 3 |
| UNDERSTAND: |  | degrees).

Two-dimensional shapes can be classified based on properties of their angles (i.e., right, acute, obtuse,) and/or properties of their line segments (i.e., parallel, perpendicular).

| KNOW: |
| :--- |
| An angle is a turn. |
| Angles are measured in degrees (i.e., 1 full |
| turn is 360 degrees, $1 / 2$ turn $=180$ degrees, $1 / 4$ |
| turn $=90$ degrees). |
| A larger angle can be decomposed into |
| smaller angles. |
| Two or more angles can be combined to make |
| a larger angle. |

Geometric measurement: understand concepts of angle and measure angles.
4.MD. 5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
b. An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees.
2-D shapes have angles at every vertex.

Perpendicular lines intersect at a 90 degree angle.

Parallel lines never intersect.

A 2-D figure has line symmetry if it can be folded along the line into matching parts.

## Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
4.G.3 Recognize a line of symmetry for a two-dimensional figure, including those found in Montana American Indian designs, as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
KEY TERMS FOR THIS PROGRESSION:
2-D shapes, Acute angle, Line, Line of symmetry, Line Segments, Obtuse angle, One-degree angle (1/360) Parallel lines, Perpendicular lines, Ray, Right triangle

## INSTRUCTIONAL STRATEGIES AND RESOURCES:

## Cluster: Geometric measurement: understand concepts of angle and measure angles.

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.

Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles.


They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.
Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler.

Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays.

Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle. Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle and circle which has a measure of 360 degrees. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

## Instructional Resources/Tools

Angle explorers
Angle ruler
Brass fasteners
Cardboard cut in strips to make an angle explorer
Protractor
Straws
Transparencies
National Council of Teachers of Mathematics. 2000-2012. Figure This: What's My Angle? Students can estimate the measures of the angles between their fingers when they spread out their hand.

Cindy Neuschwander. Sir Cumference and the Great Knight of Angleland. (Charlesbridge Publishing, 2001.) In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest, the successful completion of which will earn him his knighthood. With the help of a family heirloom that functions much like a protractor, he is able to locate the elusive King Lell and restore him to the throne of Angleland. In gratitude, King Lell bestows knighthood on Sir Radius.

Public Broadcasting Service. 1995-2012.3rd Grade Measuring Game: Identify acute, obtuse and right angles in this online interactive game
Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

## Angles:

Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle.

Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

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Symmetry:
When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.

With the use of a dynamic geometric program, students can easily construct points, lines and geometric figures. They can also draw lines perpendicular or parallel to other line segments.

Two-dimensional shapes:
Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

Informal understanding of the characteristics of triangles is developed through angle measures and side length relationships. Triangles are named according to their angle measures (right, acute or obtuse) and side lengths (scalene, isosceles or equilateral). These characteristics are used to draw triangles.

## Instructional Resources/Tools

Geoboards
GeoGebra (free software for learning and teaching)
Mirrors

## CONNECTIONS TO OTHER DOMAINS ANDIOR CLUSTERS:

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NF.4)

## STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/

DELAWARE - LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content areas/math.shtml

OHIO - INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3\&TopicRelationID=1704\&Content=134773


[^0]:    ${ }^{1}$ See Glossary, Table 2 in MCCS document.

[^1]:    ${ }^{2}$ Grade 4 expectations in this domain are limited to whole numbers less than or equal to $1,000,000$.

[^2]:    ${ }^{3}$ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

